# Practice questions: Chi-square test of independence

Test your knowledge of the [chi-square test of independence](https://www.scribbr.com/statistics/chi-square-test-of-independence/) with these practice questions. You can find the [answers and calculations](#_jvu0vfxjuuh1) here.

## Questions

### Question 1

A dog trainer wants to know if golden retrievers and French bulldogs are equally good at learning how to skateboard. She tries to train 40 golden retrievers and 60 French bulldogs to skateboard and finds the following:

|  |  |  |
| --- | --- | --- |
|  | **Skateboards** | **Can’t skateboard** |
| **Golden retrievers** | 20 | 20 |
| **French bulldogs** | 50 | 10 |

Should she reject the null hypothesis that the dog’s breed is unrelated to their skateboarding ability?

1. She should reject the null hypothesis.
2. She should fail to reject the null hypothesis.

### Question 2

A restaurant reviewer wants to know if three popular burger restaurants are equally recommended by their customers. At each of the three restaurants, he asks 25 random customers whether they would recommend the restaurant to a friend. He finds the following:

|  |  |  |
| --- | --- | --- |
|  | **Would recommend** | **Would not recommend** |
| **Tasty Burgers** | 20 | 5 |
| **Burger Prince** | 22 | 3 |
| **Burger Town** | 18 | 7 |

Should he reject the null hypothesis that the proportion of customers recommending the restaurant is the same for the three restaurants?

1. He should reject the null hypothesis.
2. He should fail to reject the null hypothesis.

## Answers

Here you can find the answers to the [practice questions](#_rhzoydocze6j).

### Answer 1

**Correct Answer = a** -She should reject the null hypothesis.

**Step 1: Calculate the expected frequencies**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Skateboards | Can’t skateboard | Row total |
| Golden retrievers | 20  **(40 \* 70) / 100 = 28** | 20  **(40 \* 30) / 100 = 12** | 40 |
| French bulldogs | 50  (**60 \* 70) / 100 = 42** | 10  **(60 \* 30) / 100 = 18** | 60 |
| Column total | 70 | 30 | *N* = 100 |

**Step 2: Calculate chi-square**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Intervention** | **Outcome** | **Observed** | **Expected** | ***O - E*** | **(*O - E*)2** | **(*O - E*)2 / *E*** |
| Golden retrievers | Skateboards | 20 | 28 | -8 | 64 | 2.29 |
| Can’t skateboard | 20 | 12 | 8 | 64 | 5.33 |
| French bulldogs | Skateboards | 50 | 42 | 8 | 64 | 1.52 |
| Can’t skateboard | 10 | 18 | -8 | 64 | 3.56 |

Χ2 = 2.29 + 5.33 + 1.52 + 3.56 = 12.7

**Step 3: Find the critical chi-square value**

Since there are two dog breed and two outcomes there is (2 - 1) \* (2 - 1) = 1 degree of freedom.

For a test of significance at α = .05 and *df* = 1, the Χ2 critical value is 3.84.

**Step 4: Compare the chi-square value to the critical value**

Χ2 = 12.7

Critical value = 3.84

The Χ2 value is greater than the critical value*.*

**Step 5: Decide whether the reject the null hypothesis**

The Χ2 value is greater than the critical value. Therefore, the dog trainer should **reject** the null hypothesis that a dog’s breed is **unrelated** to whether they can learn to skateboard. Her data suggests that a larger proportion of french bulldogs can learn to skateboard than golden retrievers.

### Answer 2

**Correct Answer = b -** He should fail to reject the null hypothesis.

**Step 1: Calculate the expected frequencies**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Would recommend | Would not recommend | Row total |
| Tasty Burgers | 20  **(25 \* 60) / 75 = 20** | 5  **(25 \* 15) / 75 = 5** | 25 |
| Burger Prince | 22  **(25 \* 60) / 75 = 20** | 3  **(25 \* 15) / 75 = 5** | 25 |
| Burger Town | 18  **(25 \* 60) / 75 = 20** | 7  **(25 \* 15) / 75 = 5** | 25 |
| Column total | 60 | 15 | 75 |

**Step 2: Calculate chi-square**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Intervention** | **Outcome** | **Observed** | **Expected** | ***O - E*** | **(*O - E*)2** | **(*O - E*)2 / *E*** |
| Tasty Burgers | Would recommend | 20 | 20 | 0 | 0 | 0 |
| Would not recommend | 5 | 5 | 0 | 0 | 0 |
| Burger Prince | Would recommend | 22 | 20 | 2 | 4 | 0.2 |
| Would not recommend | 3 | 5 | -2 | 4 | 0.8 |
| Burger Town | Would recommend | 18 | 20 | -2 | 4 | 0.2 |
| Would not recommend | 7 | 5 | 2 | 4 | 0.8 |

Χ2 = 0 + 0 + 0.2 + 0.8 + 0.2 + 0.8 = 2

**Step 3: Find the critical chi-square value**

Since there are three restaurants and two outcomes there are (3 - 1) \* (2 - 1) = 2 degrees of freedom.

For a test of significance at α = .05 and *df* = 2, the Χ2 critical value is 5.99.

**Step 4: Compare the chi-square value to the critical value**

Χ2 = 2

Critical value = 5.99

The Χ2 value is less than the critical value*.*

**Step 5: Decide whether the reject the null hypothesis**

The Χ2 value is less than the critical value. Therefore, the restaurant reviewed should **not reject** the null hypothesis the proportion of customers recommending the restaurant is **the same** for the three restaurants

Example:

In a prior example we evaluated data from a survey of university graduates which assessed, among other things, how frequently they exercised. The survey was completed by 470 graduates. In the prior example we used the χ2 goodness-of-fit test to assess whether there was a shift in the distribution of responses to the exercise question following the implementation of a health promotion campaign on campus. We specifically considered one sample (all students) and compared the observed distribution to the distribution of responses the prior year (a historical control). Suppose we now wish to assess whether there is a relationship between exercise on campus and students' living arrangements. As part of the same survey, graduates were asked where they lived their senior year. The response options were dormitory, on-campus apartment, off-campus apartment, and at home (i.e., commuted to and from the university). The data are shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **No Regular Exercise** | **Sporadic Exercise** | **Regular Exercise** | **Total** |
| **Dormitory** | 32 | 30 | 28 | 90 |
| **On-Campus Apartment** | 74 | 64 | 42 | 180 |
| **Off-Campus Apartment** | 110 | 25 | 15 | 150 |
| **At Home** | 39 | 6 | 5 | 50 |
| **Total** | 255 | 125 | 90 | 470 |

Based on the data, is there a relationship between exercise and student's living arrangement? Do you think where a person lives affect their exercise status? Here we have four independent comparison groups (living arrangement) and a discrete (ordinal) outcome variable with three response options. We specifically want to test whether living arrangement and exercise are independent. We will run the test using the five-step approach.

* **Step 1.** Set up hypotheses and determine level of significance.

H0: Living arrangement and exercise are independent

H1: H0 is false.                α=0.05

The null and research hypotheses are written in words rather than in symbols. The research hypothesis is that the grouping variable (living arrangement) and the outcome variable (exercise) are dependent or related.

* **Step 2.**  Select the appropriate test statistic.

The formula for the test statistic is:

https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/ada-reference.gifhttps://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/lessonimages/equation_image7.gif .

The condition for appropriate use of the above test statistic is that each expected frequency is at least 5. In Step 4 we will compute the expected frequencies and we will ensure that the condition is met.

* **Step 3.** Set up decision rule.

The decision rule depends on the level of significance and the degrees of freedom, defined as df = (r-1)(c-1), where r and c are the numbers of rows and columns in the two-way data table.   The row variable is the living arrangement and there are 4 arrangements considered, thus r=4. The column variable is exercise and 3 responses are considered, thus c=3. For this test, df=(4-1)(3-1)=3(2)=6. Again, with χ2 tests there are no upper, lower or two-tailed tests. If the null hypothesis is true, the observed and expected frequencies will be close in value and the χ2 statistic will be close to zero. If the null hypothesis is false, then the χ2 statistic will be large. The rejection region for the χ2 test of independence is always in the upper (right-hand) tail of the distribution. For df=6 and a 5% level of significance, the appropriate critical value is 12.59 and the decision rule is as follows: Reject H0 if c 2 > 12.59.

* **Step 4.** Compute the test statistic.

We now compute the expected frequencies using the formula,

**Expected Frequency = (Row Total \* Column Total)/N.**

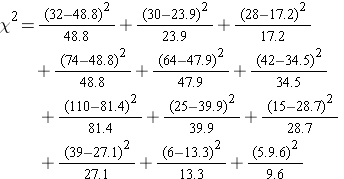
The computations can be organized in a two-way table. The top number in each cell of the table is the observed frequency and the bottom number is the expected frequency.   The expected frequencies are shown in parentheses.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **No Regular Exercise** | **Sporadic Exercise** | **Regular Exercise** | **Total** |
| **Dormitory** | 32  (48.8) | 30  (23.9) | 28  (17.2) | 90 |
| **On-Campus Apartment** | 74  (97.7) | 64  (47.9) | 42  (34.5) | 180 |
| **Off-Campus Apartment** | 110  (81.4) | 25  (39.9) | 15  (28.7) | 150 |
| **At Home** | 39  (27.1) | 6  (13.3) | 5  (9.6) | 50 |
| **Total** | 255 | 125 | 90 | 470 |

Notice that the expected frequencies are taken to one decimal place and that the sums of the observed frequencies are equal to the sums of the expected frequencies in each row and column of the table.

Recall in Step 2 a condition for the appropriate use of the test statistic was that each expected frequency is at least 5. This is true for this sample (the smallest expected frequency is 9.6) and therefore it is appropriate to use the test statistic.

The test statistic is computed as follows:

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* **Step 5.** Conclusion.

We reject H0 because 60.5 > 12.59. We have statistically significant evidence at a =0.05 to show that H0 is false or that living arrangement and exercise are not independent (i.e., they are dependent or related), p < 0.005.

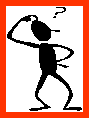
Again, the χ2 test of independence is used to test whether the distribution of the outcome variable is similar across the comparison groups. Here we rejected H0 and concluded that the distribution of exercise is not independent of living arrangement, or that there is a relationship between living arrangement and exercise. The test provides an overall assessment of statistical significance. When the null hypothesis is rejected, it is important to review the sample data to understand the nature of the relationship. Consider again the sample data.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **No Regular Exercise** | **Sporadic Exercise** | **Regular Exercise** | **Total** |
| **Dormitory** | 32 | 30 | 28 | 90 |
| **On-Campus Apartment** | 74 | 64 | 42 | 180 |
| **Off-Campus Apartment** | 110 | 25 | 15 | 150 |
| **At Home** | 39 | 6 | 5 | 50 |
| **Total** | 255 | 125 | 90 | 470 |

Because there are different numbers of students in each living situation, it makes the comparisons of exercise patterns difficult on the basis of the frequencies alone. The following table displays the percentages of students in each exercise category by living arrangement. The percentages sum to 100% in each row of the table. For comparison purposes, percentages are also shown for the total sample along the bottom row of the table.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **No Regular Exercise** | **Sporadic Exercise** | **Regular Exercise** |
| **Dormitory** | 36% | 33% | 31% |
| **On-Campus Apartment** | 41% | 36% | 23% |
| **Off-Campus Apartment** | 73% | 17% | 10% |
| **At Home** | 78% | 12% | 10% |
| **Total** | 54% | 27% | 19% |

From the above, it is clear that higher percentages of students living in dormitories and in on-campus apartments reported regular exercise (31% and 23%) as compared to students living in off-campus apartments and at home (10% each).



**Test Yourself**

**[Pancreaticoduodenectomy (PD)](javascript:void(0);)**https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/ada-reference.gif is a procedure that is associated with considerable morbidity. A [**study**](javascript:void(0);)https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/ada-reference.gif was recently conducted on 553 patients who had a successful PD between January 2000 and December 2010 to determine whether their [**Surgical Apgar Score (SAS)**](javascript:void(0);)https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/ada-reference.gif is related to 30-day [**perioperative**](javascript:void(0);)https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/ada-reference.gif morbidity and mortality. The table below gives the number of patients experiencing no, minor, or major morbidity by SAS category.

|  |  |  |  |
| --- | --- | --- | --- |
| **Surgical Apgar Score** | **No morbidity** | **Minor morbidity** | **Major morbidity or mortality** |
| 0-4 | 21 | 20 | 16 |
| 5-6 | 135 | 71 | 35 |
| 7-10 | 158 | 62 | 35 |

Question: What would be an appropriate statistical test to examine whether there is an association between Surgical Apgar Score and patient outcome? Using 14.13 as the value of the test statistic for these data, carry out the appropriate test at a 5% level of significance. Show all parts of your test.

[Answer](https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/BS704_HypothesisTesting-ChiSquare4.html#userbookmark_Answer-Pancreaticoduodenectomy)

In the [module on hypothesis testing for means and proportions](http://sph.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/), we discussed hypothesis testing applications with a dichotomous outcome variable and two independent comparison groups. We presented a test using a test statistic Z to test for equality of independent proportions. The chi-square test of independence can also be used with a dichotomous outcome and the results are mathematically equivalent.

In the prior module, we considered the following example. Here we show the equivalence to the chi-square test of independence.

Example:

A randomized trial is designed to evaluate the effectiveness of a newly developed pain reliever designed to reduce pain in patients following joint replacement surgery. The trial compares the new pain reliever to the pain reliever currently in use (called the standard of care). A total of 100 patients undergoing joint replacement surgery agreed to participate in the trial. Patients were randomly assigned to receive either the new pain reliever or the standard pain reliever following surgery and were blind to the treatment assignment. Before receiving the assigned treatment, patients were asked to rate their pain on a scale of 0-10 with higher scores indicative of more pain. Each patient was then given the assigned treatment and after 30 minutes was again asked to rate their pain on the same scale. The primary outcome was a reduction in pain of 3 or more scale points (defined by clinicians as a clinically meaningful reduction). The following data were observed in the trial.

|  |  |  |  |
| --- | --- | --- | --- |
| **Treatment Group** | **n** | **Number with Reduction**  **of 3+ Points** | **Proportion with Reduction**  **of 3+ Points** |
| **New Pain Reliever** | 50 | 23 | 0.46 |
| **Standard Pain Reliever** | 50 | 11 | 0.22 |

We tested whether there was a significant difference in the proportions of patients reporting a meaningful reduction (i.e., a reduction of 3 or more scale points) using a Z statistic, as follows.

* **Step 1.** Set up hypotheses and determine level of significance

H0: p1 = p2

H1: p1 ≠ p2                             α=0.05

Here the new or experimental pain reliever is group 1 and the standard pain reliever is group 2.

* **Step 2.** Select the appropriate test statistic.

We must first check that the sample size is adequate. Specifically, we need to ensure that we have at least 5 successes and 5 failures in each comparison group or that:

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In this example, we have

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Therefore, the sample size is adequate, so the following formula can be used:

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* **Step 3.** Set up decision rule.

Reject H0 if Z < -1.960 or if Z > 1.960.

* **Step 4.** Compute the test statistic.

We now substitute the sample data into the formula for the test statistic identified in Step 2. We first compute the overall proportion of successes:

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We now substitute to compute the test statistic.

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* **Step 5.**  Conclusion.

We reject H0 because https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/ada-reference.gifhttps://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/lessonimages/equation_image23.gif. We have statistically significant evidence at α=0.05 to show that there is a difference in the proportions of patients on the new pain reliever reporting a meaningful reduction (i.e., a reduction of 3 or more scale points) as compared to patients on the standard pain reliever.

We now conduct the same test using the chi-square test of independence.

* **Step 1.** Set up hypotheses and determine level of significance.

H0: Treatment and outcome (meaningful reduction in pain) are independent

H1:   H0 is false.         α=0.05

* **Step 2.** Select the appropriate test statistic.

The formula for the test statistic is:

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The condition for appropriate use of the above test statistic is that each expected frequency is at least 5. In Step 4 we will compute the expected frequencies and we will ensure that the condition is met.

* **Step 3.** Set up decision rule.

For this test, df=(2-1)(2-1)=1. At a 5% level of significance, the appropriate critical value is 3.84 and the decision rule is as follows: Reject H0 if χ2 > 3.84. (Note that 1.962 = 3.84, where 1.96 was the critical value used in the Z test for proportions shown above.)

* **Step 4.** Compute the test statistic.

We now compute the expected frequencies using:

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The computations can be organized in a two-way table. The top number in each cell of the table is the observed frequency and the bottom number is the expected frequency. The expected frequencies are shown in parentheses.

|  |  |  |  |
| --- | --- | --- | --- |
| **Treatment Group** | **# with Reduction**  **of 3+ Points** | **# with Reduction**  **of <3 Points** | **Total** |
| **New Pain Reliever** | 23  (17.0) | 27  (33.0) | 50 |
| **Standard Pain Reliever** | 11  (17.0) | 39  (33.0) | 50 |
| **Total** | 34 | 66 | 100 |

A condition for the appropriate use of the test statistic was that each expected frequency is at least 5. This is true for this sample (the smallest expected frequency is 22.0) and therefore it is appropriate to use the test statistic.

The test statistic is computed as follows:

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(Note that (2.53)2 = 6.4, where 2.53 was the value of the Z statistic in the test for proportions shown above.)

* **Step 5.** Conclusion.

We reject H0 because https://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/ada-reference.gifhttps://sphweb.bumc.bu.edu/otlt/MPH-Modules/BS/BS704_HypothesisTesting-ChiSquare/lessonimages/equation_image29.gif. We have statistically significant evidence at α=0.05 to show that H0 is false or that treatment and outcome are not independent (i.e., they are dependent or related). This is the same conclusion we reached when we conducted the test using the Z test above. With a dichotomous outcome and two independent comparison groups, Z2 = χ2 ! Again, in statistics there are often several approaches that can be used to test hypotheses.